

Comments on “Analytic and Numerical Study of Preheating Dynamics”

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In a recent paper Boyanovsky et al (hep-ph/9608205) studied preheating for a particular theory of the inflaton field. Although their results essentially confirm those of the original papers on reheating after inflation, they claimed that most of the earlier works on preheating did not take into account backreaction of created particles, misused the Mathieu equation, and were inconclusive about the symmetry restoration after preheating. We explain why we cannot agree with these statements.

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Recently there was an important progress in the theory of reheating of the universe after inflation. As it was shown in [1–4], in many versions of chaotic inflation the inflaton field ϕ very rapidly decays into its own quanta and other bosons due to broad parametric resonance. To distinguish this stage of explosive reheating from the stage of particle decay and thermalization, we called it *preheating*. Investigation of this process should be performed both in the broad and in the narrow resonance regime, with an account taken of backreaction of created particles and of expansion of the universe [1]. Bosons produced at that stage are far away from thermal equilibrium and have enormously large occupation numbers. Preheating leads to many interesting effects. For example, specific nonthermal phase transitions may occur soon after preheating, which are capable of restoring symmetry even in the theories with symmetry breaking on the scale $\sim 10^{16}$ GeV [5,6]. Strong deviation from thermal equilibrium and the possibility of production of superheavy particles by oscillations of a relatively light inflaton field may resurrect the theory of GUT baryogenesis [7] and may considerably change the way baryons are produced in the Affleck-Dine scenario [8].

The theory of preheating is rather complicated and strongly model-dependent. Certain aspects of this theory were investigated by many authors, and a lot of interesting information has been accumulated [1]–[16]. However, recently Boyanovsky, de Vega, Holman, and Salgado have written a paper criticizing almost all other authors studying preheating [17]. They write, in particular: “There have been a number of papers (see refs. [1,5,6,4]) dedicated to the analysis of the preheating process... In most treatments this approximation neglects all back reaction

effects. We will argue that these approaches do **not** agree with our analytical and numerical results and miss important physics... In fact, whereas the Mathieu equation has *infinitely many* forbidden bands, the exact equation has only *one* forbidden band. Such an approximation, based on the chart of the unstable bands of the Mathieu equation has been used in most [1,4], etc. treatments of preheating. These results must therefore be regarded as *highly* suspect... We also discuss why the phenomenon of symmetry restoration at preheating, discussed by various authors [5]–[12] is **not** seen to occur.”

We believe that these statements, made in [17] in bold face and in italic, are incorrect. They contradict the contents and misinterpret the papers they cited.

1. Back reaction effects

Creation of particles leads to several effects which can change the dynamics of the system. Already in our first paper [1] we did take into account backreaction of created particles, which modifies equations of motion for the interacting fields. We also took into account the contribution of the created particles to the energy density of the universe, which controls the speed of the universe expansion. Khlebnikov and Tkachev [14] studied the feedback effects of rescattering of created particles. A section of the review article by Kofman in [1] is devoted to the discussion of all of these feedback effects of created particles. Ironically, only part of the feedback effects (modification of equation of motion without an account taken of expansion of the universe and rescattering) was considered in [17].

2. The Mathieu equation

The main subject of investigation of ref. [1] was the theory of a massive inflaton field ϕ interacting with another scalar field χ . Parametric resonance in this theory is described explicitly by the Mathieu equation, and comments of ref. [17] simply do not apply to this theory.

On the other hand, the main subject of investigation of [17] was the theory $m^2\phi^2/2 + \lambda\phi^4/4$. There are two main regimes there: $\phi \ll m/\sqrt{\lambda}$ and $\phi \gg m/\sqrt{\lambda}$. The authors of [17] do not make any distinction between these two regimes because they neglect expansion of the universe. However, from [1] it follows that in expanding universe there is no parametric resonance at all for $\phi \ll m/\sqrt{\lambda}$. Therefore all results of [17] related to parametric resonance in this regime do not give a correct description of reheating in the theory $m^2\phi^2/2 + \lambda\phi^4/4$.

As for the regime $\phi \gg m/\sqrt{\lambda}$, the main statement of ref. [17] is that the parametric resonance in this model is described by Lamé equation rather than by Mathieu equation. However, this fact was already emphasized earlier in [1]. To study parametric resonance in this theory, one can approximate Lamé equation by Mathieu equation. The resonance which was found in [1] for this particular theory was only in one band, so the statement of ref. [17] that we used charts of many bands is misplaced.

Approximate treatment of Lamé equations in terms of Mathieu equation was made in [1] only as a first illustrative part of a complete investigation. It was found [1] that fluctuations approximated by Mathieu equation the unstable modes grow as $e^{3.4\mu\sqrt{\lambda}\phi t}$, where $\mu \sim 0.07$. Direct investigation of the Lamé equation reveal similar result but with $\mu \sim 0.04$, see the review by Kofman in [1]. This was the value which was used in our subsequent paper [5]. Khlebnikov and Tkachev [14] made a more complete numerical investigation of preheating in the theory $\lambda\phi^4/4$ than the investigation performed in [17], because they took into account rescattering of produced particles and expansion of the universe. They obtained the same value of μ [14]. All of these results were ignored by the authors of ref. [17].

A more general theory studied in [1] contains both the terms $m^2\phi^2/2 + \lambda\phi^4/4$ and $g^2\phi^2\chi^2$. In the limit $\phi \ll m/\sqrt{\lambda}$ decay due to parametric resonance can go only to the field χ , and there are many instability bands in this regime. As for the regime $\phi \gg m/\sqrt{\lambda}$, $g^2 \gg \lambda$, the number of instability bands for the field χ is infinitely large for all but very special relations between g^2 and λ .

It is instructive to consider the maximum value of growth index μ in the unstable bands as a function of g^2/λ . It turns out that the model considered in [17] in a certain sense is a degenerate case. The function $\mu(g^2/\lambda)$ has a minimum at $g^2/\lambda = 3$. In this case equation describing creation of χ -particles is similar to the equation for ϕ -particles in the theory $\lambda\phi^4/4$. In this particular case there is a single instability band. However, in the regime $g^2 \gg \lambda$ there are many instability bands, and the results for μ are converging to those obtained from the Mathieu equation.

3. Symmetry restoration from preheating

Finally, let us consider the theory of nonthermal phase transitions [5,6]. These phase transitions have been found in several versions of chaotic inflation theory when the inflaton field ϕ rolls down from very large values $\phi \sim M_p$. Then the oscillations of this field produces enormously large fluctuations $\langle\phi^2\rangle$ and $\langle\chi^2\rangle$, which are capable of restoring symmetry in the theory.

The authors of ref. [17] claim that they did not find any symmetry restoration from preheating. However, they studied a regime which is completely different from that of [5,6]. Instead of analysing fluctuations produced by the field rolling from $\phi \sim M_p$, they studied the theory where the field ϕ falls to the minimum of the effective potential from the vicinity of $\phi = 0$. Symmetry restoration in this case would imply return of the field ϕ exactly to the point $\phi = 0$, which is hardly possible because it loses some part of its energy for particle production. Thus it is not surprising that Boyanovsky *et al* did not find in their model the effect of symmetry restoration discovered in [5,6] in a different context.

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